

**Unit 5 In-Class review ANSWERS:**

(1) Suppose that 84% of a sample of 125 nurses working 7 AM to 3 PM shifts in city hospitals express positive job satisfaction, while only 72% of a sample of 150 nurses on 11 PM to 7 AM shifts express similar fulfillment. Establish a 90% confidence interval estimate for the difference and interpret.

**Conditions:**

- |                    |   |
|--------------------|---|
| 1. Random          | 1) Assume samples are representative of the populations   |
| 2. Independence    | 2) Nurses working day shifts and night shifts are independent of each other                             |
| 3. 10% condition   | 3) There are more than 1250 nurses working a day shift and more than 1500 nurses working a night shift. |
| 4. Success/failure | 4) 105, 108, 20, 42 $\geq 10$   |

All conditions have been met to use the Normal model for a 2 proportion z-interval.

$$(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}} = (0.84 - 0.72) \pm 1.645 \sqrt{\frac{(0.84)(0.16)}{125} + \frac{(0.72)(0.28)}{150}}$$

$$= (0.0391, 0.2009)$$

We are 90% confident that the true difference in the percent of day and night shift nurses who express positive job satisfaction is between 3.91% and 20.09%.

OR

We are 90% confident that the true proportion of nurses working a day shift who express positive job satisfaction is between 3.9% and 20.1% higher than for nurses working a night shift.

(2) A doctor thinks that less than 30 percent of all persons exposed to a certain amount of radiation will feel any ill effects. Only 9 of the 57 people exposed to such radiation felt any ill effects. Test the claim at the 0.05 level of significance.

$$H_0: p = 0.30$$

$$H_A: p < 0.30$$

**Conditions:**

- |                    |  |
|--------------------|--|
| 1. Random          | 1) assume the sample is representative of the population   |
| 2. 10% condition   | 2) There are more than 570 people exposed to the radiation |
| 3. Success/failure | 3) 17.1 and 39.9 > 10                                      |

All conditions have been met to use the Normal model for a 1 proportion z-test.

$$p = 0.30 \quad \hat{p} = \frac{9}{57} = 0.158 \quad \alpha = 0.05$$

$$z = \frac{0.158 - 0.30}{\sqrt{\frac{(0.30)(0.70)}{57}}} = -2.341$$

$$P(z < -2.341) = 0.0096$$

We reject the null hypothesis because the P-Value of 0.0096 is less than alpha of 0.05. There is statistically significant evidence that the true proportion of people exposed to this radiation that feel ill effects is less than 30%.

**(3)** Suppose in an election campaign a telephone poll of 800 registered voters under age 30 shows 460 in favor of the Republican candidate. However a second poll shows only 520 of 1000 registered voters over age 30 favored the Republican candidate. At the 1% significance level, is there sufficient evidence that the candidate's popularity is different between the two age groups?

$P_U$  – proportion of registered voters under age 30

$P_O$  – proportion of registered voters over age 30

$H_0: p_O = p_U$

$H_A: p_O \neq p_U$

**Conditions:**

- |                    |   |
|--------------------|---|
| 1. Random          | 1) Assume samples are representative of the populations   |
| 2. Independence    | 2) Registered voters over age 30 and under age 30 are independent of each other                       |
| 3. 10% condition   | 3) There are more than 8000 registered voters under 30 and more than 10000 registered voters over 30. |
| 4. Success/failure | 4) 460, 340, 520, 480 $\geq 10$   |

All conditions have been met to use the Normal model for a 2 proportion z-test.

$$\hat{p}_1 = \frac{460}{800} = 0.575 \quad \hat{p}_2 = \frac{520}{1000} = 0.520 \quad \hat{p}_{pooled} = \frac{460 + 520}{800 + 1000} = \frac{980}{1800} = 0.544$$

$$z = \frac{0.575 - 0.52}{\sqrt{\frac{(0.544)(0.456)}{800} + \frac{(0.544)(0.456)}{1000}}} = 2.328$$

$$2 \cdot P(z > 2.328) = 0.0199$$

We fail to reject  $H_0$  because the P-Value of 0.0199 is greater than alpha of 0.01. There is insufficient evidence to say that the proportion of registered voters under 30 that favor the Republican candidate is different than for registered votes over the age 30.

**(4)** In a random sample of machine parts, 18 out of 225 were found to have been damaged in shipment. Establish a 95% confidence interval estimate for the proportion of machine parts that are damaged in shipment.

**Conditions:**

- |                    |  |
|--------------------|--|
| 1. Random          | 1) stated as a random sample                           |
| 2. 10% condition   | 2) There are more than 2250 machine parts              |
| 3. Success/failure | 3) $n\hat{p} = 18 \geq 10$<br>$n\hat{q} = 207 \geq 10$ |

All conditions have been met to use the Normal model for a 1 proportion z-interval.

$$\hat{p} \pm z^* \sqrt{\frac{(\hat{p})(\hat{q})}{n}} = \frac{18}{225} \pm 1.960 \sqrt{\frac{\left(\frac{18}{225}\right)\left(\frac{207}{225}\right)}{225}} = (0.0446, 0.1155)$$

We are 95% confident that the true proportion of machine parts damaged in the shipment lies between 4.5% and 11.6%.

**(5)** Go back to Problem #2 and answer the following questions:

(a) Interpret your p-value in context of the problem

There is a 0.96% chance of getting a sample where of 15.7% or less of people exposed to the radiation feel ill effects, if the true percent of people exposed to this radiation that feel ill effects is 30%.

(b) What would a Type I error be in context?

We state that the proportion of people exposed to this radiation that feel ill effects is less than 30% when really it is not less than 30%.

(c) What would a Type II error be in context?

We state that the proportion of people exposed to this radiation that feel ill effects is not less than 30% when really it is less than 30%.

(d) What would Power be in context?

The probability that we state that the proportion of people exposed to this radiation that feel ill effects is less than 30% and it is less than 30%.

- (6) Go back to Problem #1. Using your interval, does there appear to be a difference between the percent of nurses expressing positive job satisfaction during each shift?

Since 0 is not within the interval there is strong evidence of a difference between the proportion of nurses expressing positive job satisfaction between the two shifts.

- (7) Going back to problem #4: We want to estimate the true percent of defective machine parts with a 3% margin of error and 92% confidence. How many machine parts would we have to sample? Use the value sample proportion given in Problem #4 as a good estimate for the true proportion.

$$ME = 0.03 \quad z^* = 1.751 \quad \hat{p} = \frac{18}{225} = 0.08$$

$$0.03 = 1.751 \sqrt{\frac{(0.08)(0.92)}{n}}$$

$$n = 251$$

- (8) I want to estimate the number of people who will respond "YES" to my survey question. I want a margin of error of 6% with 99% confidence. How many people do I need to sample?

$$ME = 0.06 \quad z^* = 2.576 \quad \hat{p} = ? = 0.50$$

$$0.06 = 2.576 \sqrt{\frac{(0.50)(0.50)}{n}}$$

$$n = 461$$

- (9) I have a confidence interval that is (0.42, 0.57).

(a) What is my sample proportion?

$$\hat{p} = 0.495$$

(b) What is my margin of error?

$$ME = 0.075$$

(c) If my sample size was 60, what is my level of confidence?

$$0.075 = z^* \sqrt{\frac{(0.495)(0.505)}{60}}$$

$$z^* = 1.162 \quad \rightarrow \quad CL = 75.5\%$$

- (10) Going back to Problem #4, explain what 95% confidence means in context.

About 95% of random samples of size 225 will produce confidence intervals that contain the true proportion of machine parts damaged in the shipment.