Unit 4 Exam Review Answers

1. 5 – 10%; 2 – 20%; 2 – 30%; 1 – 50%

X	10%	20%	30%	50%
P(X = x)	5/10	2/10	2/10	1/10

   a. P(>20%) = P(30%) + P(50%) = 2/10 + 1/10 = 0.30

   b. P(<20%) = P(10%) = 5/10 = 0.50

   c. $P(50\% \cap 50\%) = \left(\frac{1}{10}\right)^2 = 0.01$

   d. $P(50\%^C \cap 50\%^C \cap 50\%^C) = \left(\frac{9}{10}\right)^3 = 0.729$

   OR: binomial B(3, 0.1) P(X = 0) = 0.729

   e. $P(30\%^C \cap 30\%^C \cap 30\%^C \cap 30\%^C \cap 30\%^C \cap 30\%) = \left(\frac{8}{10}\right)^5 \cdot \frac{2}{10} = 0.0655$

   f. $1 - P(50\%^C \cap 50\%^C \cap 50\%^C \cap 50\%^C \cap 50\%^C) = 1 - \left(\frac{9}{10}\right)^5 = 0.4095$

   OR: binomial B(5, 0.1) $P(X \geq 1) = 1 - P(X \leq 0) = 0.4095$

   g. Both are incorrect. Each roll of the rubber cube is independent of any other roll. So the probability of getting a 50% discount is the same no matter what the previous values were.

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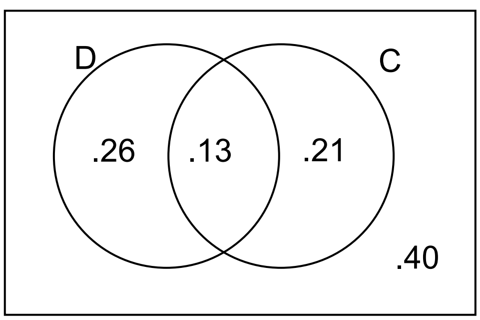
2. Dogs and cats

   a. D = Family owns at least one dog
   C = Family owns at least one cat



(.39 + .34) - .60 = .13

   b. $P(C^C \cap D^C) = 0.40$

   c. $P(C \cap D) = 0.13$

   d. $P(C|D) = \frac{P(C \cap D)}{P(D)} = \frac{0.13}{0.39} = 0.33$

   e. No. $P(C \cap D) = 0.13 \neq 0$. A household can own a cat and a dog at the same time.

   f. Yes. Knowing that a family has a dog doesn't change the probability that they own a cat. P(C) = 0.34; P(C|D) = 0.33

3. a. P(X = 3) = 0.28

   b. $P(X \leq 3) = 0.16 + 0.22 + 0.28 = 0.66$

c. $P(1<X\leq 4) = 0.22 + 0.28 + 0.20 = 0.70$

d. $E(X) = 1(0.16) + 2(0.22) + 3(0.28) + 4(0.20) + 5(0.14) = 2.94$
$Var(X) = (1 - 2.94)^2(0.16) + (2 - 2.94)^2(0.22) + (3 - 2.94)^2(0.28) + (4 - 2.94)^2(0.20) + (5 - 2.94)^2(0.14)$
$Var(X) = 1.6164$
$SD(X) = 1.2714$

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4. (a) P(A U B) = 0.65 + 0.23 – 0.15 = 0.73

(b) $P(B|A) = \frac{0.15}{0.65} = 0.2307$

(c) No. P(A n B) is not 0

(d) Possibly. P(B|A) is very close to P(B). Justify whatever answer you give!!

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5. P(D U C) = P(D) + P(C) = 0.78

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6. P(K n R) = P(K)*P(R) = 0.1633

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7. (a) P(F n H) = P(H|F) * P(F) = 0.0429

(b) P(F U H) = P(F) + P(H) – P(F n H) = 0.5671

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8. P(A U B) = P(A) + P(B) – P(A n B)
0.78 = 0.25 + P(B) – 0.12
P(B) = 0.65

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9. P(R) = 0.37
P(R n U) = 0.15

$P(U|R) = \frac{0.15}{0.37} = 0.405$

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10. P(M) = 0.5
P(M n J) = 0.20

$P(J|M) = \frac{0.20}{0.50} = 0.40$

		Breakfast		
		Yes	No	
Sex	Male	66	66	132
	Female	125	74	199
		191	140	331

11.

a. P(F) = 199/331 = 0.6012

b. P(B) = 191/331 = 0.5770

c. P(F ∩ B) = 125/331 = 0.3776

d. P(B|F) = 125/199 = 0.6281

e. P(F|B) = 125/191 = 0.6545

f. No it doesn't appear that they are independent. Knowing that a student is female changes the probability that they ate breakfast. P(B|F) = 0.6281 ≠ P(B) = 0.5770

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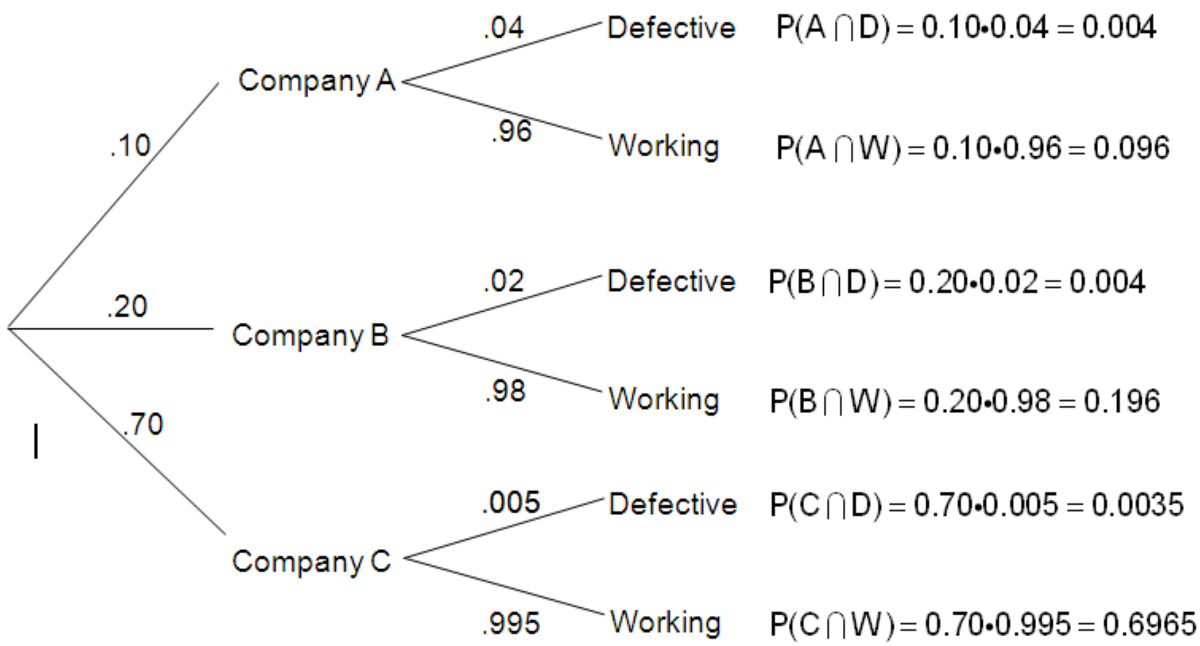
12. Tree Diagram



$$P(B|D) = \frac{P(B \text{ I } D)}{P(A \text{ I } D) + P(B \text{ I } D) + P(C \text{ I } D)} = \frac{0.004}{0.004 + 0.004 + 0.0035} = 0.348$$

13. Probability model for the payout of a prize

X = payout	-\$2	\$73	\$148	\$248	\$498
P(X=x)	1496/1500	1/1500	1/1500	1/1500	1/1500

For (b) and (c), just put X-values in L1, probabilities in L2 and then do 1-var stats L1, L2:

b. E(X) = -\$1.35

c. SD(X) = \$15.06

d. E(X + X + X) = -1.35 + -1.35 + -1.35 = -\$4.05

SD(X + X + X) = $\sqrt{15.06^2 + 15.06^2 + 15.06^2}$ = \$26.08

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14. SAT Scores

M – score for a randomly selected male E(M) = 1532 SD(M) = 312

F – score for a randomly selected female E(F) = 1506 SD(F) = 304

a. M + F;

E(M + F) = 1532 + 1506 = 3038

SD(M + F) = $\sqrt{312^2 + 304^2} = 435.61$

b. M – F

E(M – F) = 1532 – 1506 = 26

SD(M – F) = $\sqrt{312^2 + 304^2} = 435.61$

c. Assume a normal model with N(26, 435.61).

Female scored higher than male → P(F > M) → P(M -- F < 0)

P(M –F < 0) = normalcdf(-E99, 0, 26, 435.61) = 0.4762

There is a 47.62% probability that a randomly selected female scored higher than a randomly selected male on the SAT.

15. Random Variables

	Mean	SD
X	12	5
Y	18	8

a. -2X
E(-2X) = -2(12) = -24
SD(-2X) = 2(5) = 10

b. 4Y – 7
E(4Y – 7) = 4(18) – 7 = 65
SD(4Y – 7) = 4(8) = 32

c. X + Y
E(X + Y) = 12 + 18 = 30
SD(X + Y) = $\sqrt{5^2+8^2}=9.43$

d. X – Y
E(X – Y) = 12 – 18 = -6
SD(X – Y) = $\sqrt{5^2+8^2}=9.43$

e. $X_1 + X_2$
$E(X_1 + X_2) = 12 + 12 = 24$
$SD(X_1 + X_2) = \sqrt{5^2+5^2}=7.07$

f. 2X – 4Y
E(2X – 4Y) = 2(12) – 4(18) = -48
E(2X – 4Y) = $\sqrt{2^2(5^2)+4^2(8^2)}=33.53$

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16. Check: Bernoulli?
1. 2 Outcomes – has jumper cables or doesn't have jumper cables
2. p = 0.40
3. p is constant
3. 10% Condition – we can assume independence as long as the number of drivers asked is less than 10% of all drivers.

a. Define Y = yes, someone can jump your car
Define N = no, someone cannot jump your car
P(N ∩ N ∩ N ∩ N ∩ N ∩ N ∩ Y) = $(0.60)^6(0.40)$ = 0.0187

b. P(X < 7) = P(X ≤ 6) = P(X = 1)+P(X=2) + P(X=3) + P(X=4) + P(X=5) + P(X=6) = 0.9533

c. Geom(0.40)
E(X) = $\frac{1}{p}=\frac{1}{0.40}=2.5$ drivers

d. B(8,0.40)
P(X = 3) = binompdf(8, 0.4, 3) = 0.2787

e. B(6,0.40)
$P(X\geq 4)=1-P(X\leq 3)=1-binomcdf(6,0.4,3)=0.1792$

f. B(10,0.40)
$P(X\geq 6)=1-P(X\leq 5)=1-binomcdf(10,0.4,5)=0.1662$

g. B(12,0.40)
E(X) = np = 12(0.40) = 4.8 drivers

h. B(80,0.40)
CHECK: Success/Failure Condition

$np = 80(0.40) = 32 \geq 10$

$nq = 80(0.60) = 48 \geq 10$

Since there were at least 10 expected successes and 10 failures we can use the normal model to approximate the binomial probabilities.

N(32,4.382)

P(X ≤ 30) = normalcdf(-E99, 30, (80*0.40), $\sqrt{(80 * 0.40 * 0.60)}$) = 0.324