Testing Hypotheses About Proportions

Procedure for Hypothesis Tests

Step 1: Hypotheses

- Write the null hypothesis. This is denoted as H_0 and read as "H not". Remember that a hypothesis is about an unknown population parameter, so be sure to use the proper statistical notation: for example, p = 0.50.
- Write the alternative hypothesis. This is denoted as H_a. Decide whether you need a one-tail or two-tail test. If one tail is it the upper or lower tail we are interested in?
- Note that the null hypothesis is that NOTHING has changed (status quo).

Step 2: Plan

- Decide what inference procedure is called for. Right now this is easy – we are just looking for a one-proportion z-test. This will become a key issue when we expand our repertoire of hypothesis tests. Means or proportions? One group or two? Independent or matched?
- List the assumptions and check the conditions:
  - SRS
  - 10% condition
  - np and nq <=10
- Name the test. Indicate that because the conditions are satisfied it is okay to use a Normal model and that you will perform (for now) a one-proportion z-test.

Step 3: Mechanics

- Using the proper notation, write down the statistics. For this type of test those are: sample size, the observed number of successes and the sample proportion.
- Draw a curve showing the sampling model. Mark the hypothesized parameter and the observed statistic and shade the appropriate tail (s) to show the region corresponding to the P-value.
- Calculate the value of the test statistics – here it is z, later it might be t or χ^2. Show the formula and substitute all of the proper values and give the final result. You may use a calculator to check the results you got be hand, but it is still necessary to show all of the work.

$$z = \frac{\hat{p} - p}{SE(\hat{p})}$$

Remember: $$SE(\hat{p}) = \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

- Find the critical value for the level of significance given. We will use P-value method later.

Step 4: Conclusion

- Link the critical value to a decision.
- State the decision about the null hypothesis. Either reject or do not reject.
- Interpret this decision in the proper context.

The Null and Alternative hypotheses will take on one of these forms:

$H_0 : p = .6$
$H_a : p < .6$ You think the proportion has decreased. (one-tailed to the left)

$H_0 : p = .6$
$H_a : p > .6$ You think the proportion has increased. (one-tailed to the right)

$H_0 : p = .6$
$H_a : p \neq .6$ You think the proportion has changed or is different. (two-tailed test)

Example 1:
A 1996 report from the US Consumer Product Safety Commission claimed that 90% of all American homes have at least one smoke detector. A city's fire department has been running a public safety campaign about smoke detectors with posters, billboards, and ads on radio and TV and in the newspaper. The city wonders if the effort has raised the local level about the 90% national rate. Building inspectors visit 400 randomly selected homes and find that 376 have detectors. Is this strong evidence that the local rate is higher than the national rate?

Example 2:
There are supposed to be 20% purple M&M's. Suppose a bag of 122 has only 21 of the purple ones. Does this contradict the company's 20% claim? This a two-tail test since we are equally interested to discover that there are too many.