Hypothesis Testing for One Sample Mean with Known Population Standard Deviation

How is this different from One Proportion Z Test?
Assumptions:

1. The sample is a SRS.
2. The population is normal or n >=30.
3. The population standard deviation is known.

Test Statistic:

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{n}}$$

Everything else is the same…

Example 1:
In a quality control situation, the mean weight of objects produced is supposed to be 16 ounces with a standard deviation of 0.4 ounces. A random sample of 70 objects yields a mean weight of 15.8 ounces. Is it reasonable to assume that the production standards are being maintained? Use a level of significance of 0.05.

Example 2:

The nicotine content in milligrams (mg) in cigarettes of a certain brand is normally distributed with mean μ and population standard deviation 0.1 mg. The brand advertises that the mean nicotine content of its cigarettes is 1.5 mg, but measurements on a random sample of 100 cigarettes of this brand gave a mean of $\bar{x} = 1.53\ mg$. Is this evidence that the mean nicotine content is actually higher than advertised? Test at the 5% significant level.

Example 3:
A simple random sample of 10 people from a normal population has a mean age of 27. Can we conclude that the mean age of the population is not 30? The population variance is known to be 20. Let the level of significance be 0.05.

Type I and Type II Error

		Population truth	
		H_o true	H_o false
Decision based on sample	Reject H_o	Type I Error (∝)	Correct Decision (1- β)
	Do Not Reject H_o	Correct Decision	Type II Error (β)

Let's talk about this in terms of the justice system. If you convict an innocent person, then you have committed a Type I error. If you let a guilty person go you are committing a Type II error. We try to minimize Type I errors by demanding unanimous jury guilty verdicts. In civil suits, however, many states try to minimize Type II errors by simply accepting majority verdicts.

The significance level or ∝ is the probability of committing a Type I error. The probability of committing a Type II error is β. The POWER of a test is the probability that a Type II error is NOT committed and is designated by 1- β. In other words, power is the probability of rejecting the null hypothesis when it is in fact false.

What things affect power?

1. If all other things are held constant, then as α increases, so does the power of the test. This is because a larger α means a larger rejection region for the test and thus a greater probability of rejecting the null hypothesis. That translates to a more powerful test. The price of this increased power is that as α goes up, so does the probability of a Type I error should the null hypothesis in fact be true.
2. A larger sample size n will reduce the standard deviations, making both graphs narrower resulting in smaller α, smaller β, and larger power.
3. The greater the difference in the null hypothesis p_o and the true value p, the smaller the risk of a Type II error and the greater the power.

We choose alpha, so why not choose a very, very small value so we eliminate Type I errors? Well….because of the relationship between alpha and beta. If we choose a very small value of alpha it makes it very difficult to reject the null hypothesis and more Type II errors will occur.

Example:
Consider a scenario regarding online gaming customers. A sales manager was interested in deciding if the proportion of males was less than the 0.75 that was claimed by the marketing manager. The appropriate hypotheses for this situation are:

H_o: $p = 0.75$
H_a: $p < 0.75$

a) Identify the Type I error in this scenario and provide a possible consequence of this error.

b) Identify the Type II error in this scenario and provide a possible consequence of this error.

c) What can be done to increase the power of this test?