Confidence Intervals for Proportions -- Chapter 19

Using a measurement from a sample, we are never able to say exactly what a population proportion or mean is; rather we always say we have a certain **confidence** that the population proportion or mean lies in a particular **interval**. The particular interval is centered around a sample proportion or mean (or other sample statistic) and can be expressed as the sample estimate plus or minus an associated **margin of error**.

Using what we know about sampling distributions, we are able to establish a certain confidence that a sample proportion or mean lies within a specified interval around the population proportion or mean. Typically we consider 90%, 95% and 99% confidence intervals, but any percent is possible.

There are two aspects of a confidence interval:

1. CONFIDENCE INTERVAL is generally expressed as:

   estimate or sample statistic $\pm$ margin of error

2. There is a success rate for the method, that is, the proportion of times repeated applications of this method would capture the true population parameter. For example, if we have a 95% confidence interval for a sample mean of heights in our class. Remember we took 24 samples of 4 heights in class. We would create a confidence interval for each sample (you will see how to do that in a moment). We are confident that 95% of these intervals contain the true population mean.

HOW TO CONSTRUCT A CONFIDENCE INTERVAL

1. Determine the sample statistic.
2. Select a confidence level. This value generally will be given to you. It measures the uncertainty of the sampling method.
3. Find the MARGIN OF ERROR. The general equation for margin of error is:

   margin of error = critical value * sample standard deviation
4. Specify the confidence interval.

Ok….we, have actually already done this before. Let's do a practice problem doing it the way we know then we will apply this concept of confidence interval.

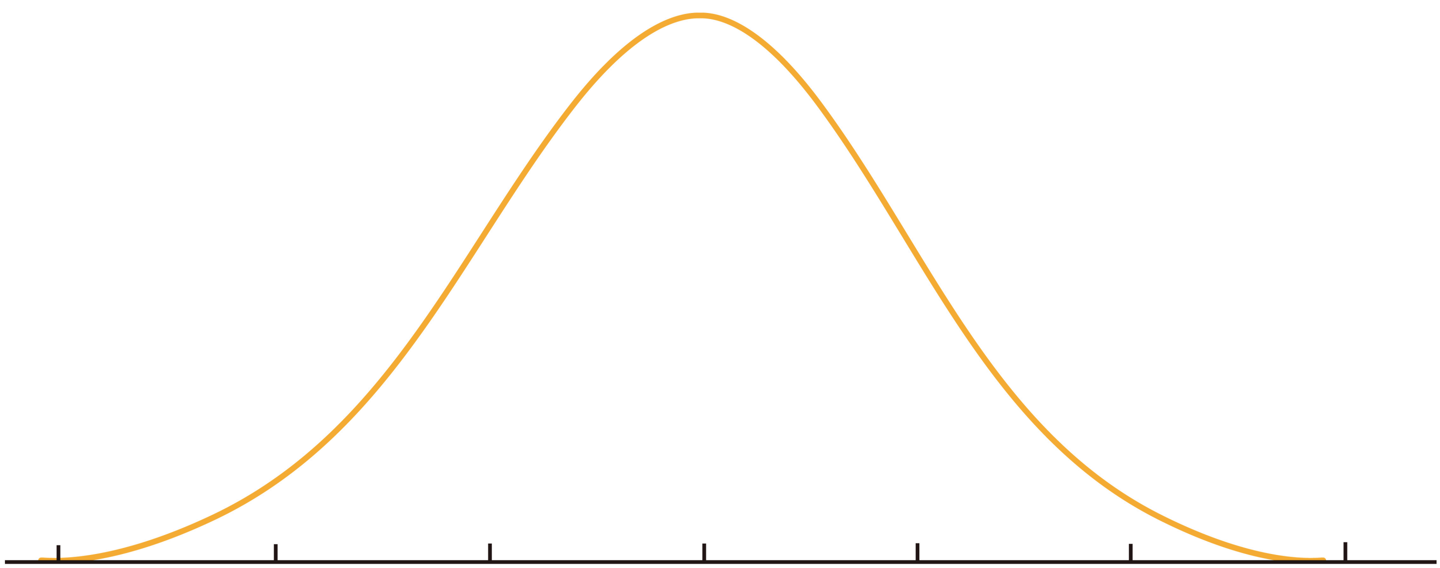
EXAMPLE:
In a simple random sample of machine parts, 18 out of 225 were found to have been damaged in shipment. Establish a 95% confidence interval estimate for the proportion of machine parts that are damaged in the shipments.

What do we know already from sample distributions (with proportions)?

Well…the unbiased estimate of the population proportion is p which is 18/225 or 0.08. We check the assumptions for normality:

- np = 18 > 10 and nq = 207 > 10
- It is reasonable to assume that 225 is less than 10% of all shipped parts.

The standard deviation is $\sigma_p = \sqrt{\frac{pq}{n}}$. In this case it is $\sigma_p = \sqrt{\frac{(.08).92)}{225}} = .0181$



OF COURSE! You can do it on your calculator as well.

CALCULATOR:

STAT<TESTS<1-PropZInt. In this example x = 18, n = 225 and C-Level is .95.

Why is it a little bit different than using the empirical rule? The 68-95-99.7 rule rounds a bit. So, in actuality 95% is within 1.96 standard deviations if you use the table.

BACK to our question….HOW DO WE GET THE CRITICAL VALUE?

For large samples (n > 30) we will use z scores. So, for this problem our critical value is 1.96.

To show the equation:

95% CI = $.08 \pm 1.96 * .0181 = (.04455, .11545)$

We are 95% certain that the proportion of parts that are damaged in the shipment are between the two above values.

TRY:

1000 randomly selected Americans were asked if they believed the minimum wage should be raised. 600 said yes. Construct a 95% confidence interval for the proportion of Americans who believe that the minimum wage should be raised.

TRY #2: Do the sample problem above with a 90% CI and a 99% confidence interval.

HOMEWORK #5

Do the following problems:

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

6) Find the critical value $z_{\alpha/2}$ that corresponds to a degree of confidence of 98%. 6) ________

A) 2.575 B) 2.05 C) 2.33 D) 1.75

7) Find the critical value $z_{\alpha/2}$ that corresponds to a degree of confidence of 91%. 7) ________

A) 1.75 B) 1.70 C) 1.645 D) 1.34

Express the confidence interval in the form of $\hat{p} \pm E$.

8) $-0.134 < p < 0.666$ 8) ________

A) $\hat{p} = 0.266 - 0.4$ B) $\hat{p} = 0.4 \pm 0.5$ C) $\hat{p} = 0.266 \pm 0.5$ D) $\hat{p} = 0.266 \pm 0.4$

Solve the problem.

9) The following confidence interval is obtained for a population proportion, p: (0.700, 0.728) 9) ________

Use these confidence interval limits to find the point estimate, $\hat{p}$.

A) 0.718 B) 0.714 C) 0.728 D) 0.700

10) The following confidence interval is obtained for a population proportion, p: 10) ________

$0.478 < p < 0.510$

Use these confidence interval limits to find the margin of error, E.

A) 0.032 B) 0.016 C) 0.494 D) 0.017

Find the margin of error for the 95% confidence interval used to estimate the population proportion.

11) $n = 151$, $x = 122$ 11) ________

A) 0.110 B) 0.00201 C) 0.0628 D) 0.0565

12) In a survey of 4800 T.V. viewers, 50% said they watch network news programs. 12) ________

A) 0.0141 B) 0.0106 C) 0.0162 D) 0.0185

Use the given degree of confidence and sample data to construct a confidence interval for the population proportion p.

13) $n = 54$, $x = 13$; 95 percent 13) ________

A) $0.144 < p < 0.338$ B) $0.126 < p < 0.356$
C) $0.145 < p < 0.337$ D) $0.127 < p < 0.355$

14) $n = 140$, $x = 81$; 90 percent 14) ________

A) $0.514 < p < 0.644$ B) $0.509 < p < 0.649$
C) $0.510 < p < 0.648$ D) $0.513 < p < 0.645$