Confidence Intervals Part II

Some review from Part I:
CI #1: Confidence Intervals for Proportion (1-proportion z-interval)

The general formula is:

$$\hat{p} \pm z^* * \sqrt{\frac{\hat{p}(\hat{q})}{n}}$$

Where:

- Point estimate or statistic = $\hat{p}$
- Margin of Error (ME) = $z^* * \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
- Standard Error (SE) = $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
- Critical Value = z^*

The conditions that need to be met are:
1. The sample is a SRS (Simple Random Sample).
2. The sample is < 10% of the population.
3. The sampling distribution of the sample proportion is approximately normal. np≥ 10 and nq≥ 10.

Let's do an example.
The owner of a popular chain of restaurants wishes to know if completed dishes are being delivered to the customer's table within one minute of being completed by the chef. A random sample of 75 completed dishes found that 60 were delivered within one minute of completion. Find the 95% confidence interval for the true population proportion.

Step 1: Identify the population of interest and define the parameter of interest being estimated.

Step 2: Identify the appropriate CI by name or formula.

Step 3: Verify any conditions (assumptions) that need to be met for CI.

Step 4: Calculate the CI.

Step 5: Interpret the CI in the context of the situation.

CI #2: Confidence Interval for Means with σ known (z-interval).

The general formula is:

$$\bar{x} \pm z^* * \frac{\sigma}{\sqrt{n}}$$

where:

- Point estimate or statistic = $\bar{x}$
- Margin of Error (ME) = $z^* * \frac{\sigma}{\sqrt{n}}$
- Standard Error (SE) = $\frac{\sigma}{\sqrt{n}}$
- Critical Value = z^*

The conditions that must be met are:

1. The sample is a SRS.
2. The population is normal or $n \geq 30$
3. The population standard deviation is known.

Example:

An asbestos removal company places great importance on the safety of their employees. The protective suits that the employees wear are designed to keep asbestos particles off the employee's body. The owner is interested in knowing the average amount of asbestos particles left on the employee's skin after a days work. A random sample of 100 employees had skin tests after removing their protective suit. The average number of particles found was .481 particles per square centimeter. Assuming the population standard deviation is 0.35 particles per square centimeter, calculate a 95% CI for the number of particles left on the employee's skin.

Step 1: Identify the population of interest and define the parameter of interest being estimated.

Step 2: Identify the appropriate CI by name or formula.

Step 3: Verify any conditions (assumptions) that need to be met for CI.

Step 4: Calculate the CI.

Step 5: Interpret the CI in the context of the situation.

CI #3: Confidence Interval for Means with σ unknown (t-interval).

The general formula is:

$$\bar{x} \pm t^* * \frac{s}{\sqrt{n}}$$

where:

- Point estimate or statistic = $\bar{x}$
- Margin of Error (ME) = $t^* * \frac{s}{\sqrt{n}}$
- Standard Error (SE) = $\frac{s}{\sqrt{n}}$
- Critical Value = t^*
- Degrees of Freedom (df)= n-1

TABLE D

t **distribution critical values**

	Upper-tail probability *p*											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z^*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291

The conditions that must be met are:

1. The sample is a SRS.
2. Using a value of s to estimate the population standard deviation.
3. Population distribution is given as normal OR $n > 30$ OR $15 < n < 30$ with normal probability plot showing little skewness and no extreme outliers OR $n < 15$ with npp showing no outliers and no skewness. In general, a boxplot is the easiest way graphically to show possible normality.

Example:

A biology student at a major university is writing a report about bird watchers. She has developed a test that will score the abilities of a bird watcher to identify common birds. She collects data from a random sample of people that classify themselves as bird watchers (data below). Find a 90% CI for the mean score of the population of bird watchers.

4.5	9.1	8	5.9	7.0	5.2	7.3	7.0	6.6	5.1	7.6	8.2	6.4	4.8	5.8
6.2	8.5	7.3	7.8	7.4										

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Homework:

Free Response (show all steps):

1. A random survey finds that 587 out of 675 adults claim never to take naps during the day. Calculate a 95% CI of the proportion of adults who never take naps.

2. Mrs. Harrison keeps a record of the gas mileage for her car each time she fills up. Five recent visits yielded the following results (in mpg):

   20.35 21.26 19.58 18.76 20.56

   Find the 90% CI for the true mean gas mileage of her car.

4. A 2009 Harris poll found that 33% of 2,681 Americans surveyed said they were very happy. Find a 95% CI for the percent of Americans who are very happy.

5. From a population with standard deviation 25, a sample of size 100 is drawn. The mean of this sample is 235. Find the 99% confidence interval of the mean of the population.