Comparing Two Populations or Treatments

Objectives:

- Correctly set up and carry out a hypothesis test for the difference in two population means using independent or paired samples.
- Construct and interpret a confidence interval for a difference of two means.

Independent vs Paired Samples:

- Independent samples are samples where knowing the individuals selected for one sample does not tell you anything about which individuals are selected for the other sample.
- With paired samples, an observation from one sample is paired in a meaningful way with an observation in the other sample (many times it comes from the same person).

So…first questions to ask:

1. Independent or paired?
2. Population standard deviation known or not?

Type I: Difference of two population means using independent samples.

Step 1: Assumptions or conditions

1. Two samples are independent.
2. Both population standard deviations are known (z) or unknown (t).
3. Both sample sizes are greater than or equal to 30 OR we can show normality (unimodal and relatively symmetric).
4. Both samples are SRS.

Step 2: Null and alternative hypotheses.

- Null is generally $H_o = \mu_1 - \mu_2 = 0$
- Alternative can be >, < or not equal to.

Step 3: Critical Value (z)

Step 4: Test statistic

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

or

and df = n_1-1 or n_2-1 whichever is smaller OR use your calculator. Just make sure you specify what df you are using.

Step 5: Conclusion

Sample Problem 1:
Students want to see if regular and low-fat chocolate chip cookies have the same number of chocolate chips in a cookie, on average. They suspect the way low-fat cookies are made is by simply reducing the number of chocolate chips in each cookie. To test this theory, they selected random samples of cookies of each type and counted the number of chocolate chips in each cookie. Summary statistics are shown in the table below. Boxplots of the two samples were approximately symmetric and there were no outliers.

Cookie	Sample Size	Mean Chips	Standard Deviation
Regular	25	16.3	2.29
Low-Fat	25	15.2	3.25

Based on the data provided, is there evidence that the mean number of chips for regular cookies is greater than the mean number for low-fat cookies?

Pooled T-Test:
The pooled t test is used when it is known that the two population variances are equal. However, this is rarely known. For the AP exam, it is advised that you always use the unpooled test when testing hypotheses about the difference in means using independent samples.

Type II: Difference between two population means using paired samples.

If the sample sizes are the same, think about whether or not the samples are paired. If the sample sizes are not the sample, then it cannot be a paired test. However if they are the same, it does not necessarily indicate they are paired.

Step 1:
Assumptions:
1. The samples are paired and therefore not independent.
2. Both samples are SRS.
3. The sample size is large (greater than or equal to 30) or can assume normality.
4. Population standard deviation is unknown (t).

Step 2: Hypothesis
- $H_0: \mu_d = 0$ where μ_d is difference in mean
- Alternate hypothesis is < > or not equal to.

Step 3: Critical Value

Step 4: Test statistic

$$t = \frac{\bar{x}_d - 0}{\frac{s_d}{\sqrt{n}}}$$ where df = n – 1

Step 5: Conclusion

Sample Problem 2:

Twenty five students each counted the number of chocolate chips in a randomly selected regular chocolate chip cookie and recorded the counts as seen in the table below. Prior to counting, each student flipped a coin to decide if he or she would count the regular or low-fat cookie first. Is there evidence that, on average, regular chocolate chip cookies contain more chips than low-fat cookies?

Person	R (regular)	L (low-fat)	Difference
1	20	17	
2	17	13	
3	12	10	
4	15	13	
5	16	12	
6	20	23	
7	15	16	
8	19	18	
9	18	16	
10	17	15	
11	15	19	
12	18	18	
13	13	12	
14	21	14	
15	17	11	
16	13	16	
17	16	16	
18	17	16	
19	15	15	
20	18	20	
21	14	12	
22	13	15	
23	21	19	
24	12	10	
25	15	13	

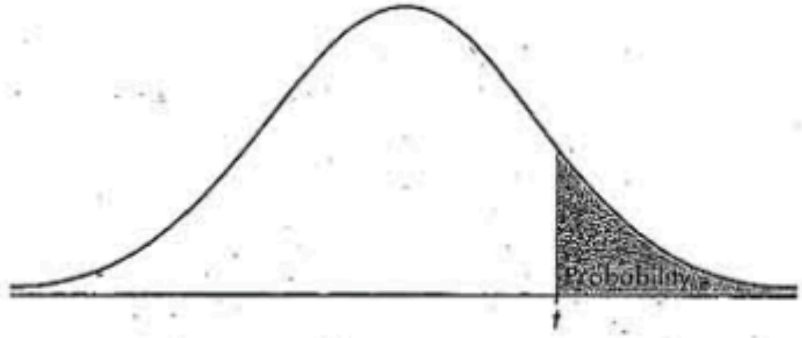

TABLE B: *t*-DISTRIBUTION CRITICAL VALUES

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%

Confidence Interval:

$$(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$