Chapter 16 – Random Variables Continued

Some review…..

Expected Value of a discrete random variable

$$\mu = E(x) = \sum x * P(x)$$

Variance of a discrete random variable

$$\sigma^2 = Var(X) = \sum (x - \mu)^2 * P(x)$$

Standard deviation of a discrete random variable

$$\sigma = SD(X) = \sqrt{Var(X)}$$

Example:

Policyholder Outcome	Payout	Probability
Death	10,000	1/1000
Disability	5,000	2/1000
Neither	0	997/1000

a) Find the expected value of the insurance payout.

b) Find the standard deviation of the insurance payout.

Example 2:

As the head of inventory for Knowway computer company, you were thrilled that you had managed to ship 2 computers to your biggest client on the day the order arrived. You are horrified, though, to find out that someone had restocked refurbished computers in with the new computers in your storeroom. The shipped computers were selected randomly from the 15 computers in stock, but 4 of these were actually refurbished.

If your client gets 2 new computers, things are fine. If the client gets one refurbished computer, it will be sent back at your expense -- $100 – and you will replace it. However, if both computers are refurbished, the client will cancel the order this month and you will lost $1000. What is the expected value and the standard deviation of your loss?

Independence vs Dependence

An automobile salesman sells three models of vans with selling prices of $20,000, $25,000 and $30,000 respectively. For each sale, the salesman receives a bonus of either $500 or $750. The probabilities of the various outcomes are given by the following table:

	500	750
20,000	.30	.05
25,000	.20	.20
30,000	.05	.20

If X is the amount of sale (row values) and Y is the bonus amount, what is the probability distribution of X?

Of Y?

What is the probability of a $500 bonus given that the sale was $30,000?

Are they independent? Remember if independent $P(A \cap B) = P(A) * P(B)$ or $P(A|B) = P(A)$

Example:

In a study on vitamin C and the common cold, people were assigned to take either no vitamin C or 500 mg daily, and it was noted whether or not they came down with a cold. Letting X, the vitamin C random variable take values 0 and 500 and Y (colds variable) take 0 for no cold or 1 for at least 1.

	0	1
0	.10	.30
500	.15	.45

Are they independent? We have to check each cell…

Example:
Suppose set X = {2, 9, 11, 22} and set Y = {5, 7, 15} What is the mean and standard deviation of each?

Suppose set Z is the set of differences X – Y (subtracting each element of Y from each element of X). Find this set. Find the mean and standard deviation of this set and compare to above.

Some rules:

RULE 1: We know that adding or subtracting a constant changes the mean by that constant but does not change the standard deviation. The same is true of random variables.

$$E(X \pm c) = E(X) \pm c$$
$$Var(X \pm c) = Var(X)$$

RULE 2: Multiplying the dataset by a constant multiplies the mean by that constant and the variance by the square of that constant.

$$E(aX) = aE(X)$$
$$Var(aX) = a^2 Var(X)$$

RULE 3: The expected value of the sum/difference of two random variables is the sum/difference of the expected values. The same is true of the variance.

$$E(X \pm Y) = E(X) \pm E(Y)$$
$$Var(X \pm Y) = Var(X) + Var(Y)$$

Example:
Given independent random variables with means and standard deviations as shown, find the mean and standard deviation of each of these variables.

	Mean	SD
X	10	2
Y	20	5

a) 3X

b) Y+6

c) X+Y

d) X – Y

Homework #1: p 383 #1-10, 24 a-d